Bremsstrahlung from Electrons and Positrons in Peripheral Relativistic Heavy Ion Collisions

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Abstract

We study the spectrum of the bremsstrahlung photons coming from the electrons and positrons, which are produced in the strong electromagnetic fields present in peripheral relativistic heavy ion collisions. We compare different approaches, making use of the exact pair production cross section in heavy ion collisions as well as the double equivalent photon approximation.

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I. INTRODUCTION

The cross-section for electron-positron pair production in relativistic heavy ion collisions is huge, see, e.g., [1] for a general reference, due to the coherence of all the protons in each nucleus. The e^+e^--pairs are produced in an interaction with two quasi-real photons. On the other hand, the cross-section for bremsstrahlung in peripheral relativistic heavy ion collisions was found to be small, both for real [2] and virtual [3] bremsstrahlung photons. This is essentially due to the large mass of the heavy ions. Since the cross section for e^+e^- pair production is so large, of the order of 100 kbarn for RHIC and LHC, one can expect to see sizeable effects from the bremsstrahlung radiation of these light mass particles. It is the purpose of this paper to calculate the cross-section for bremsstrahlung due to this effect and show its main characteristics. The general theoretical framework is presented in Sec. II. Numerical results along with a discussion are given for RHIC and LHC conditions in Sec. III, Sec. IV contains our conclusions.

II. THEORETICAL FRAMEWORK

We consider peripheral collisions where the ions do not interact strongly with each other (so called "peripheral collisions"). The basic mechanism to produce e^+e^--pairs is the two-photon mechanism. In principle the soft bremsstrahlung photons can be emitted from the two heavy ions, as well as from the produced lepton pair. Due to their large mass, we can neglect the emission from the heavy ions (see, for example, [2] and [3]) and we calculate the bremsstrahlung due to the dileptons only. This is shown in Fig. 1. In the low energy limit (infrared (IR) limit) the photon emission from the external lines is dominant over the emission from internal lines. This is a well known general result, see, e. g., [4], and the cross section for soft photon emission of the process

$$Z + Z \rightarrow Z + Z + e^+ + e^- + \gamma \tag{1}$$

can be calculated as

$$\frac{d\sigma}{d^3p_+ d^3p_- d\Omega d\omega} = -e^2 \left[\frac{p_-}{p_- k} - \frac{p_+}{p_+ k} \right]^2 \frac{\omega^2}{4\pi^2 \omega} \frac{d\sigma_0}{d^3p_+ d^3p_-}, \tag{2}$$

where σ_0 denotes the cross section for the e^+e^- pair production in heavy ion collisions. Using the code for the exact lowest order cross section for the e^+e^- pair production in external field approximation of [5], we can calculate the soft photon emission from the outgoing lepton lines numerically according to this equation. Inclusive bremsstrahlung cross sections are then calculated by integrating over the unobserved electron pairs

$$\frac{d\sigma}{d\Omega d\omega} = \int \frac{d\sigma}{d^3p_+ d^3p_- d\Omega d\omega} d^3p_+ d^3p_-. \tag{3}$$

Eq. (2) is only valid as long as the energy and momentum of the bremsstrahlung photon is small compared to the other energies and momenta involved in the process. Following the discussion in [6], we therefore use the following restriction condition:

As a measure of the influence of the additionally emitted photon, we use the invariant mass of the lepton pair with and without the photon emission. This has the advantage of being a Lorentz scalar. We expect the low energy approximation to be valid if

$$(p_+ + p_- + k)^2 - (p_+ + p_-)^2 \ll (p_+ + p_-)^2, \tag{4}$$

which can be rewritten conveniently as

$$2(p_+ + p_-) \cdot k \ll M_{e+e-}^2, \tag{5}$$

where M_{e+e-} denotes the invariant mass of the lepton pair. For a given value of the photon momentum k this is a restriction on the values of the momenta p_+ and p_- of the lepton pair. In the numerical calculation, we keep only those processes, where the above estimate is fulfilled as an inequality.

The majority of the leptons are produced with an invariant mass of the order of 2–5 m_e, see, e.g., [5,7]. Therefore this restriction is not important for photon energies which are much smaller than 1 MeV. The momentum of the pair $(p_+ + p_-)$ is also almost aligned along the beam axis in most cases. As a function of the angle with the beam axis, the scalar-product $(p_+ + p_-) \cdot k$ will therefore be smaller for small angles and larger for large angles. Therefore we expect that this condition is more important for photons emitted under a large angle from the beam axis. In the following, we call this *"method 1"* to calculate bremsstrahlung cross section.

An alternative approach, to be called *"method 2"*, is possible by using the DEPA (double equivalent photon approximation). The two heavy ions are accompanied by a spectrum of equivalent (quasireal) photons. The collision of these photons leads to

$$\gamma + \gamma \rightarrow e^+ + e^- + \gamma. \tag{6}$$

We calculate the cross-section for this subprocess in lowest order QED using a computer algebra program [8]. This cross-section is then folded with the equivalent photon numbers $n(\omega)$, which describe the probability of an equivalent photon to be emitted by one of the ions leading to

$$\frac{d\sigma}{d^3p_+ d^3p_- d\Omega d\omega} = \int \frac{d\omega_1}{\omega_1} \int \frac{d\omega_2}{\omega_2} n(\omega_1) n(\omega_2) \frac{d\sigma(\gamma + \gamma \rightarrow e^+ e^- + \gamma)}{d^3p_+ d^3p_- d\Omega d\omega} \tag{7}$$

This method has the advantage that the kinematics of the bremsstrahlung emission is treated correctly. There is no restriction on the photon energy, as is the case for the IR method (eqs 2–5). Of course this calculation can only be as good as the underlying DEPA used to calculate the $e^+ e^-$ pair production.

In addition, even if the DEPA reproduces the total pair production (only) cross section reasonably well, this cannot be expected automatically for the bremsstrahlungs cross section, as both processes are expected to be sensitive to different areas of the $e^+ e^-$ phase space. For the equivalent photon spectrum we use (see, e.g., [2,9])

$$n(\omega) = \frac{2Z^2\alpha}{\pi} [\xi K_0(\xi) K_1(\xi) - \frac{1}{2} \left[K_1^2(\xi) - K_0^2(\xi) \right], \tag{8}$$

where the K_i are the McDonald functions, $\xi = \omega R/\gamma$, and R is a suitably chosen cutoff parameter. For rather soft pairs — which are of importance here — one may choose R to be of the order of the Compton wavelength of the electron $\lambda_c = 1/m_e$ [2,9].

As *"method 3"* we have also used the low energy (IR) approximation for the cross-section $d\sigma(\gamma + \gamma \rightarrow e^+e^-\gamma)$ in eq. 7. We use eqs. (2)–(5), where $d\sigma_0$ in eq. (2) is the cross section for the process $\gamma + \gamma \rightarrow e^+ + e^-$.

In order to see what range of applicability the IR approximation has, we have compared the cross section for eq. (6) using the exact calculation to the IR method (eqs. (2)–(5)). We go to the photon-photon center of mass frame and perform calculations for different values of the invariant mass. We integrate over the momenta of the two unobserved leptons. In this case $d\sigma_0$ denotes the cross-section for the process $\gamma + \gamma \rightarrow e^+ + e^-$. Figure 2 shows the cross section $d\sigma/d\Omega d\omega$, where θ is the angle between the bremsstrahlung photon and one of the initial photons. The energy of the bremsstrahlung photon was chosen to be $\omega = 1$ MeV. One clearly sees that the IR approximation is very good for large invariant masses compared to the bremsstrahlung photon energy. At invariant masses comparable to the bremsstrahlung photon energy, on the other hand, we find disagreement, as expected. We also see that the deviation between the exact QED calculation and the IR approximation is stronger for larger angle, which is the kind of behavior we expect from the discussion of the restriction condition (see eqs. (4) and (5)).

III. NUMERICAL RESULTS

In the following we use the three different methods as described above in order to calculate the bremsstrahlung spectrum for the heavy ion case. Comparing the different calculations allows us to check both, the validity of the low energy approximation together with our restriction condition by comparing method 2 and 3, but also the validity of the DEPA (by comparing method 1 and 3), which depends on the choice of the cutoff parameter R. The comparison of the three different results are shown for the condition at RHIC (Au-Au collisions, $\gamma = 100$) and LHC (Pb-Pb collisions, $\gamma = 3400$) in Figs. 3 and 4. The two calculations using the DEPA (method 2 and 3) are in agreement with each other within a factor of two, justifying the use of the IR approximation (together with the restriction condition (eq. (5)). (Without this condition they would differ by more than a factor of 100 at 3 MeV and 90^0.)

Comparing them with the calculation using the exact pair production (method 1), one finds that using R equal to the Compton wavelength of the electron, as is usual done, gives cross section, which are too small. Therefore we have adjusted its value to get better agreement between them. We get "fair" agreement when using $R \approx 100$fm. The results in Figs. 3 and 4 are therefore calculated with this cutoff. A single R does not seem to be able to reproduce the results for all angles at the same time. This situation is similar to the one of pair production with large transverse momenta (see [10]). An improvement, which introduces a R that depends on the final state, is "work in progress" and will be addressed in a future publication.

We have compared our results also with the analytic expressions as given in [11]. In this reference, photon emission at large angles was studied for e^+e^- collisions. Their approach can be carried over to the heavy ion case with the appropriate modifications. For relatively

low (equivalent) photon energies, the equivalent photon spectrum of eq. (8) is given approximately by

$$n(\omega) \approx \frac{2Z^2\alpha}{\pi} \ln \left(\frac{\gamma}{\omega R} \right). \tag{9}$$

For $\ln(\gamma) \gg \ln(\omega R)$, one obtains

$$n(\omega) \approx \frac{2Z^2\alpha}{\pi} \ln (\gamma) \tag{10}$$

This can be compared to the expression for the equivalent photon spectrum used in ref [11]. In the reference, the expression

$$n(\omega) = \frac{Z^2\alpha}{\pi} L \tag{11}$$

is used, with $L = 2\ln(2\gamma) = \ln(s/m_e^2)$ (see their eq. (3)). We find the modification of the equivalent photon spectrum due to the factor ωR to be important in the heavy ion case (see below). Due to the low mass of the electrons γ-values at electron colliders are much higher than at heavy ion colliders (e.g., at RHIC we have $\gamma \approx 100$, whereas $\gamma \approx 200000$ at LEP ($E_{el} = 100\text{GeV}$)).

For $K_\perp = \omega \sin(\theta) \gg m_e$ an analytic formula is given (eq. (5) of [11]):

$$\frac{d\sigma}{d\Omega d\omega} = \frac{4Z^4\alpha^5}{\pi^3 K_\perp^4} L^2 \left[\frac{7}{12} L_0 - C \right] \omega \tag{12}$$

with $C \approx 0.03$ and $L_0 = \ln(K_\perp^2/m_e^2)$. For $\omega \ll m_e$ one finds (eq. (4) of [11])

$$\frac{d\sigma}{d\Omega d\omega} = \frac{Z^4\alpha^5}{2\pi^3 m_e^2} \frac{L^2}{\omega \sin^2(\theta)} \left[\frac{128\pi^2}{108} - 6 \right] \tag{13}$$

This equation contains the famous $1/\omega$-dependence of the bremsstrahlung cross-section. The spectrum therefore extends down to very low energies, well into the region of visible light. Therefore it is worthwhile to note, that these photons might also be of interest to measure/"view" the interaction region at heavy ion colliders.

In Fig. 5 we compare our calculations for $\theta = 90^0$ with the one using their equivalent photon spectrum and also with their analytic expressions eqs. (12) and (13). One sees a definite discrepancy between the calculations with the different equivalent photon spectra. This is due to the γ values at the heavy ion colliders, which are not high enough to justify the neglect of the ωR term in the equivalent photon spectra (see the discussion following eq. (11) above). One can see also that our calculation with the simplified version eq. (11) of the equivalent photon spectrum is in very good agreement with the analytical results for high and low energies of [11].

We also note that the analytical formulae (eqs. (12) and (13)) show a different angular dependence for the high and low energy limits. For the low energies (eq. (12)) there is a $1/(\sin^2\theta)$ dependence, for the high energies (eq (12)) a steeper $1/\sin^4\theta$ dependence. This trend is also visible in our calculations. For larger angles a $1/(\sin^2\theta)$ dependence is seen

for bremsstrahlungs photon energies below about 1 MeV, a $1/(\sin^4 \theta)$ dependence for larger energies.

It seems interesting to compare these formulae to the ones for bremsstrahlung due to the heavy ions (see Ch 5.1 of [2]). These formulae contain a factor $Z^6 \alpha^3 / M_A^2$. In the present case we have a scaling factor $Z^4 \alpha^5 / m_e^2$ (or, for higher energies $Z^4 \alpha^5 / \omega^2$). As was already discussed qualitatively in the introduction, the radiation from the electron is more important than the one from the heavy ions; this can be seen directly from these scaling factors.

IV. CONCLUSION

In this paper, we have studied a new type of bremsstrahlung process in peripheral relativistic heavy ion collisions. We have shown that this is the dominant mechanism for bremsstrahlung. We have used three different methods to calculate the corresponding bremsstrahlung spectra. There is general agreement between these methods in the IR region. We expect the method 1 to be the most reliable for the soft photon region. For hard spectra, method 2 is applicable, as it does not rely on the soft photon approximation; but it depends on the validity of the double equivalent photon approximation (DEPA). The DEPA was found to be dependent on the cutoff parameter R, which is theoretically not too well defined. We find fair agreement between the different approaches by using an R, which is somewhat smaller than conventional wisdom predicts. Method 2 will be most useful for future calculations of the harder part of the bremsstrahlung spectrum.

These low energy photons constitute a background for relativistic heavy ion colliders. Unlike the copiously produced low energy electrons and positrons, they are of course not bent away by the magnets and could be a hazard for the detectors. Recently the soft bremsstrahlung photons from central ultrarelativistic nucleus-nucleus collisions were suggested to be used to infer the rapidity distribution of the outgoing charge [12]. The presently considered soft photons from peripheral collisions could be a source of background for the considered experiment.
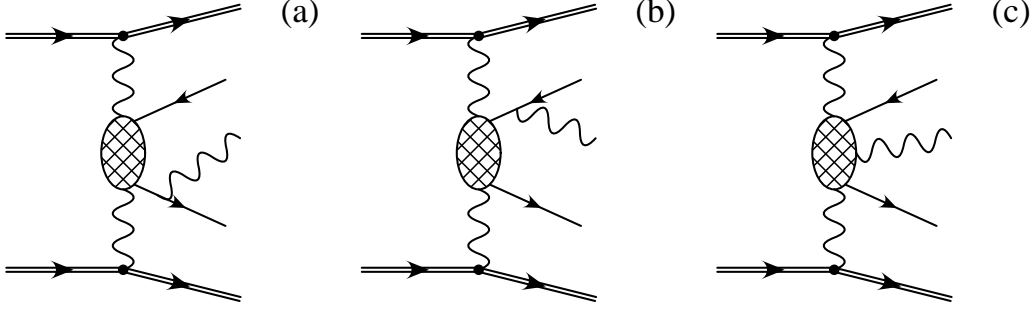
FIG. 1. *The Feynman graphs considered in this paper for the emission of bremsstrahlung photons in peripheral heavy ion collisions. In the IR limit, graphs (a) and (b) are dominant.*
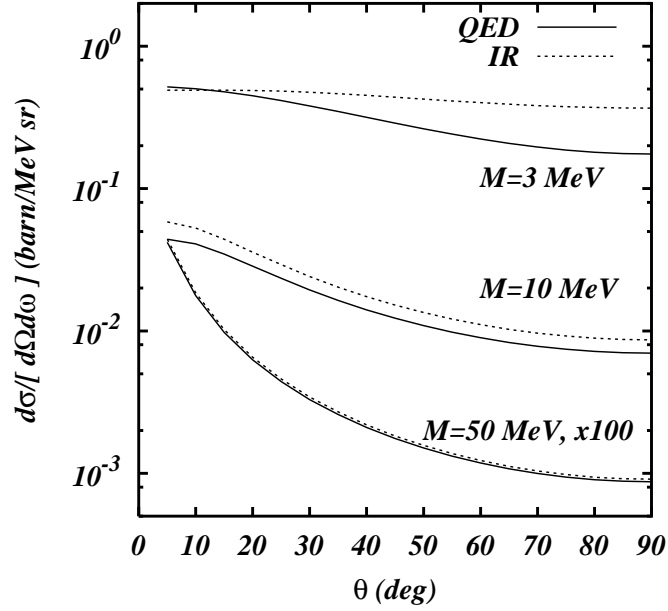

FIG. 2. The inclusive differential cross section $d\sigma/d\Omega d\omega$ (see eq.3) for the process(eq.6) is shown for different fixed photon-photon invariant masses and in the photon-photon rest frame. θ is the angle between the bremsstrahlungs photon and one of the initial photons. The energy of the bremsstrahlung photon is ω=1 MeV. The solid line are the lowest order QED results, whereas the dotted line those of the IR approximation.
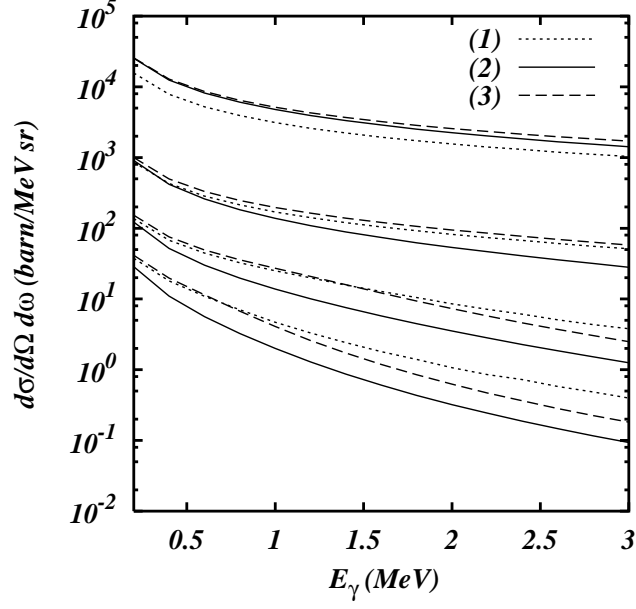
FIG. 3. The inclusive differential cross section $d\sigma/d\Omega d\omega$ is shown for RHIC conditions (γ=100, Au-Au, Z=79). Results are shown for photon energies up to 3 MeV and for different angles: 1, 10, 30, 90^0 from top to bottom. The cutoff parameter R was set to 100fm. The different calculations ((1)–(3)) are as explained in the text.
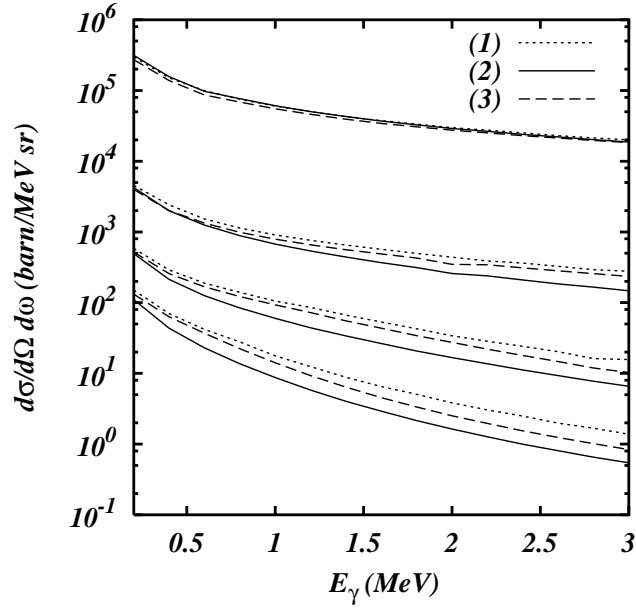

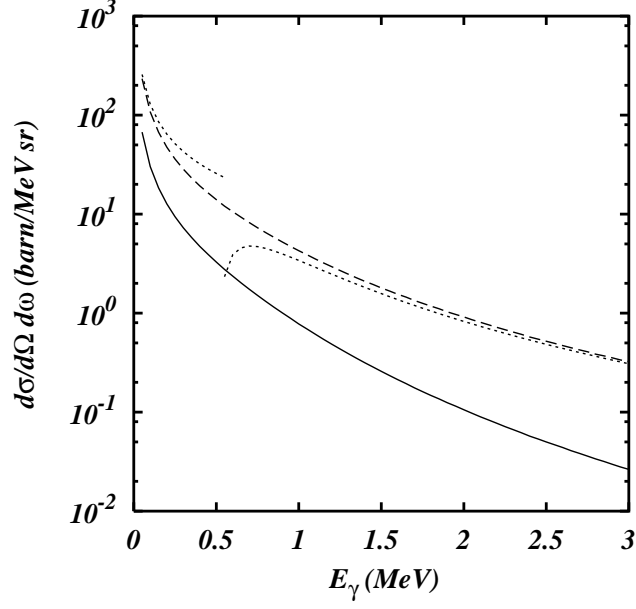FIG. 4. Same as figure above, but now for the LHC (γ=3400, Pb-Pb, Z=82).

FIG. 5. The differential cross section $d\sigma/d\Omega d\omega$ is shown for RHIC (gamma=100,Au-Au,Z=79) conditions and for $\theta = 90^0$. The calculations are done according to method 2. The solid line is the calculation with the equivalent photon spectrum as given in eq. (8). The dashed line uses the simplified equivalent photon spectrum eq. (11). The dotted lines are the analytic expressions of eq. (4) and (5) of [11] for low and high energies respectively (see eqs 12 and 13).

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